Multiple Beta Spectrum Analysis Method Based on Spectrum Fitting

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Introduction

◆ **Beta Spectrum Analysis**

- Beta spectrum has a wide range of energy distribution by beta-decay with neutrino.
- When the sample of several mixed radioactive nuclides is measured, it is difficult to divide each nuclide due to the overlapping of spectrums.
- The analysis of the beta spectrum based on the curve fitting, is suggested to separate mixed beta ray source.

Methods

◆ **Concept of Beta Spectrum Separation**

- Spectral analysis of one nuclide is performed based on the maximum energy of the measured mixed beta spectrum.
- Define the spectrum of each nuclide by excluding the defined spectrum from the entire spectrum.
- Repeat this procedure to define all nuclides spectrum.

Methods

◆ Basic assumption

- Most different beta nuclides has different maximum energy except for ^{129}I (E_{max} = 0.154 MeV) and ^{14}C(E_{max} = 0.156 MeV) which has overlapped beta spectrum each other as shown in Table 1.

- The maximum energy(E_{max}) of beta nuclides was compared by applying the concept of relative difference between two nuclides.

- Relative difference = (higher energy - lower energy) / higher energy * 100

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Methods

◆ **Basic assumption**

- Spectral separation among the mixed spectrums with different beta nuclides can be implemented from the comparison between maximum energy and energy less than 5% of the maximum energy.

- All beta nuclides represented in Table 1 showed a relative difference of more than 5%.

Table 1 The maximum energy per major beta nuclides

	E_{max} (MeV)	difference	Relative difference		E_{max} (MeV)	difference	Relative difference
^{3}H	0.01859	0.04841	72.25	^{90}Sr	0.546	0.141	20.52
^{63}Ni	0.067	0.089476	57.18	^{85}Kr	0.687	1.022	59.80
^{14}C	0.156476	0.010524	6.30	^{32}P	1.709	0.573	25.11
^{35}S	0.167	0.127	43.19	^{90}Y	2.282	1.258	35.54
^{99}Tc	0.294	0.252	46.15	^{106}Rh	3.540		

Methods

◆ Data Preparation

- Referring to ICRU, the information about the count ratio per the energy of ^{32}P, ^{90}Y and ^{106}Rh was used.

- The count ratios and energies of the each radionuclide were **interpolated** because the given energy value according to a type of radionuclides is different.

- Fig. 1 shows count ratio per the energy of each radionuclide. The blue hexagram is ^{32}P, red pentagram is ^{90}Y and yellow diamond is ^{106}Rh.

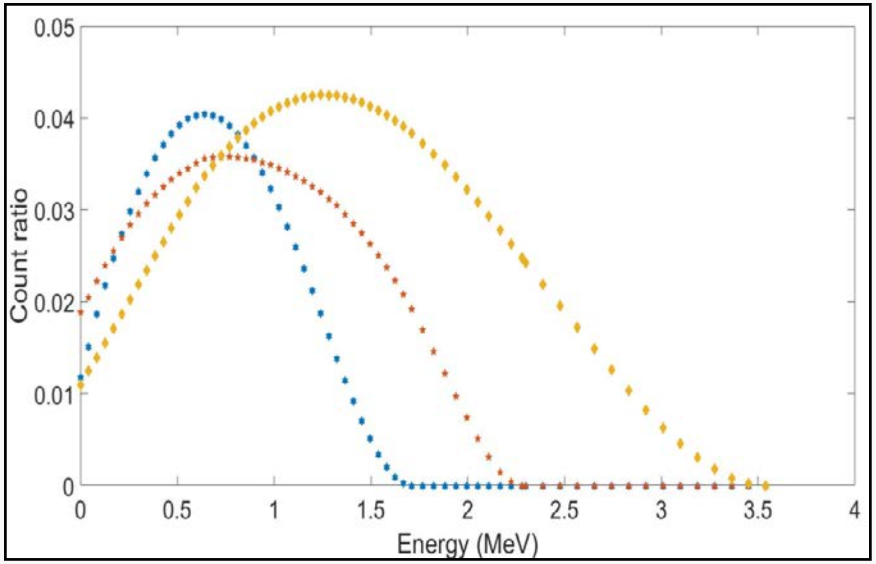

Fig. 1. Prepared interpolated spectrum data for ^{32}P, ^{90}Y and ^{106}Rh

Methods

◆ Data Preparation

- This research assumes that the count number per specific energy is proportional in accordance with the intensity of radioactivity.

- The count number of mixed nuclides is defined as sum of the count number per specific energy of each nuclide.

- The data was prepared for three divided cases.
  1) The radioactivity of one of the radioactive nuclides is very strong.
  2) The radioactivity of one of the radioactive nuclides is very small.
  3) The values of the radioactivity of all the radioactive nuclide are similar.

Methods

◆ Data Preparation

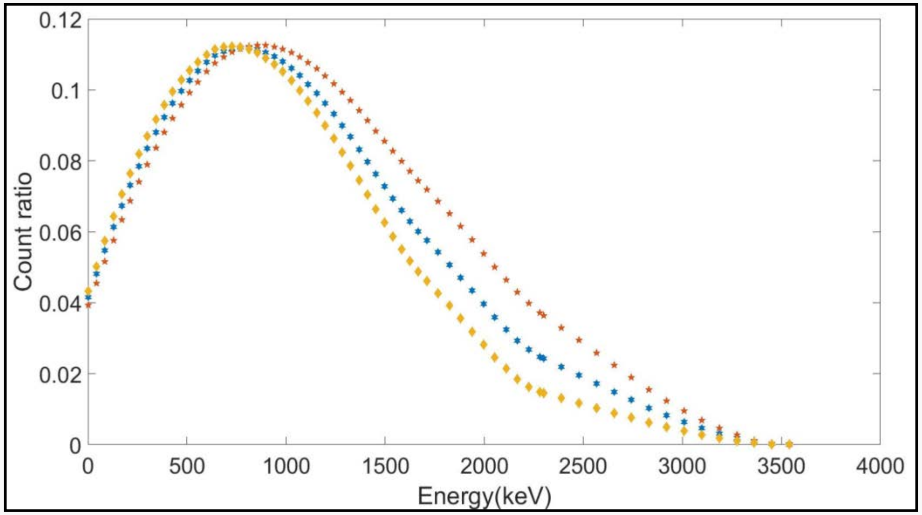

Fig. 2. Mixed spectrum for different mixing ratio

- Fig. 2. The ratio of nuclides (^{32}P : ^{90}Y :^{106}Rh) is 1:1:1: in blue hexagram, 1:1:2 in red pentagram and 2:2:1 in yellow diamond.
- The shape of spectrum varies with the **mixing ratio** of nuclides.
- Therefore, these data is used for application of proposed beta spectrum analysis method.

Methods

◆ Curve Fitting Method

- The functions for the spectral shape due to each radionuclide were derived by using the prepared data on count rate per specific energy to simulate the spectrum of each nuclide.

- The curve fitting was performed using commercially available program MATLAB software.

- Fourier, polynomial, Gaussian and sum of sine methods are used for candidate functions to make beta spectrum.

- Also, the **highest order** of fitting method is used for more precise application result.

Methods

◆ Curve Fitting Method

- The used fitting method and order is Fourier-8, polynomial-9, Gaussian-8 and sum of sine-8.
- Gaussian fitting method shows poor application results in overall data. Also the fitting results of Fourier and polynomial methods have poor performance at end region of spectrum.
- Therefore, **sum of sine fitting** method is processed to select suitable fitting method.

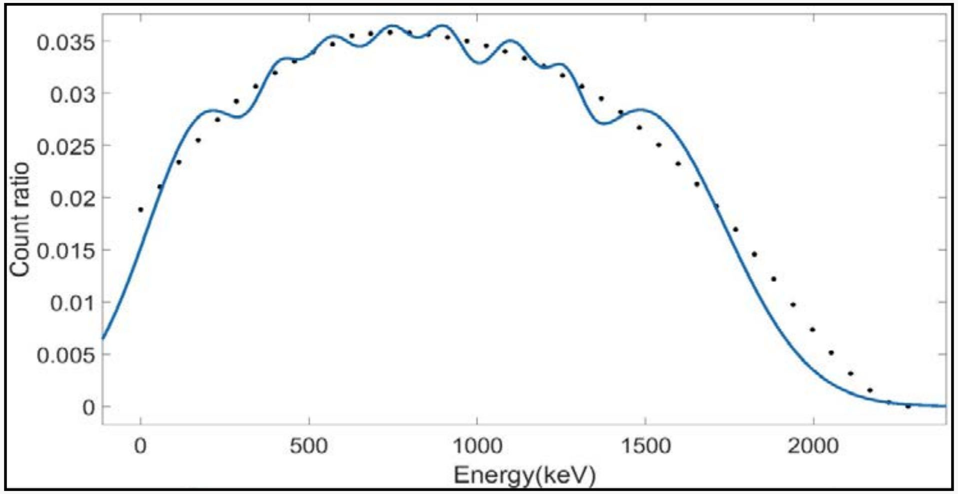

Fig. 3. Application result of Gaussian fitting method for ^{90}Y spectrum data

Methods

◆ **Nuclide definition and Separation**

- The beta nuclides that the **maximum energy** is different due to each kind of beta radionuclides. Therefore, the maximum energy of the mixed spectrum is used for separation.

- The count rate of E_{max} and E_{max-10} channel is assumed that the result is generated by single radionuclide.

- The number of **count per specific energy** (C_E) is assumed that it has proportional relationship with the value of multiplying count ratio and activity.

$$C_E = \mathrm{A} * CR_E$$

A : Radioactivity
$\mathrm{CR_E}$: Count ratio at energy E

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Results

◆ Sum of sine fitting method

- $Y = a_1\sin(b_1E+C_1) + a_2\sin(b_2E+C_2) + a_3\sin(b_3E+C_3) + a_4\sin(b_4E+C_4) + a_5\sin(b_5E+C_5) + a_6\sin(b_6E+C_6) + a_7\sin(b_7E+C_7) + a_8\sin(b_8E+C_8)$

- Fig. 4 shows the application result of sum of sine fitting method in case of ^{90}Y.

- Black point is the count ratio per specific energy of ICRU data and blue line is the result of sum of fitting method. The fitting result is good agreement with reference data.

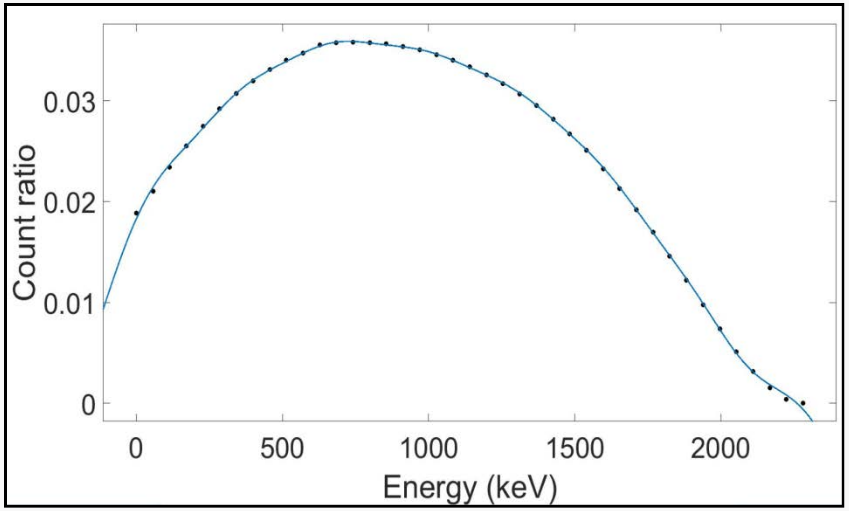

Fig.4. Application result of sum of sine fitting method for ^{90}Y spectrum data

Results

Table 2 Constants and R^2 for the fitted function equation of each nuclide

	^{32}P	^{90}Y	^{106}Rh
a_1	0.03891	0.03955	0.03887
b_1	0.001807	0.001353	0.0008726
c_1	0.165	0.2609	0.1572
a_2	0.007272	0.005739	0.008922
b_2	0.003622	0.002707	0.001749
c_2	0.3526	1.321	0.2465
a_3	0.001474	0.001353	0.0008175
b_3	0.007243	0.005417	0.003493
c_3	0.9177	0.6792	1.413
a_4	0.0005915	0.0007281	0.0005408
b_4	0.01087	0.00813	0.005243
c_4	1.156	1.244	1.578
a_5	0.0003062	0.0004385	0.0004028
b_5	0.01451	0.01085	0.006995
c_5	1.316	1.326	1.803

Results

Table 2 (cont.) Constants and R^2 for the fitted function equation of each nuclide

	^{32}P	^{90}Y	^{106}Rh
a_6	0.0001813	0.0002396	0.0002467
b_6	0.01817	0.01359	0.008759
c_6	1.441	1.486	1.803
a_7	0.0001201	0.0001777	0.0001553
b_7	0.02187	0.01637	0.01055
c_7	1.519	1.731	1.572
a_8	9.37e-05	0.0001611	0.0001154
b_8	0.02568	0.01922	0.01241
c_8	1.515	1.537	1.369
R^2	0.9999	0.9997	0.9999

- Table 2 shows constants and R^2 for the equation of each nuclide
- R^2 is the **coefficient of determination**

Results

Table 3: The comparison result of ICRU reference and application result of sum of sine fitting method

	E_{max}		
	ICRU	Fitted	Difference
^{32}P	0	-0.0004358	0.0004358
^{90}Y	0	-0.0005993	0.0005993
^{106}Rh	0	-0.0004992	0.0004992
	E_{max-10}		
	ICRU	Fitted	Difference
^{32}P	0.01626	0.01625	1.0438E-05
^{90}Y	0.01920	0.01919	1.3114E-05
^{106}Rh	0.01488	0.01488	1.9279E-06

Results

◆ Curve Fitting Method

- Table 3 shows comparison result of ICRU reference and application result of sum of sine fitting method.

- $\boldsymbol{E_{max}}$ is the count rate ratio value at the maximum energy count and $\boldsymbol{E_{max-10}}$ is the value at 10^{th} before maximum energy.

- The sum of sine fitting method shows good performance.

- Therefore, sum of sine fitting method is applied for spectrum analysis for mixed beta nuclide source.

Results

◆ Process of separtion

- First process is the definition of the ratio of ^{106}Rh by using the **ratio of** $\boldsymbol{E_{max-10}}$ value.

- Next step is multiplying the ratio of E_{max-10} value to the spectral function of ^{106}Rh and deducting the value of count per specific energy from mixed spectrum data.

- Mixed spectrum of ^{32}P, ^{90}Y is obtained.

- Same process is repeated for ^{90}Y.

- Finally all the nuclides are defined and **separated** from multiple beta spectrums.

Results

◆ Process of separtion

- The blue spectrum is mixed spectrum of ^{32}P, ^{90}Y and ^{106}Rh.
- The green spectrum is mixed spectrum of ^{32}P, ^{90}Y.
- The red spectrum is a spectrum of ^{32}P.

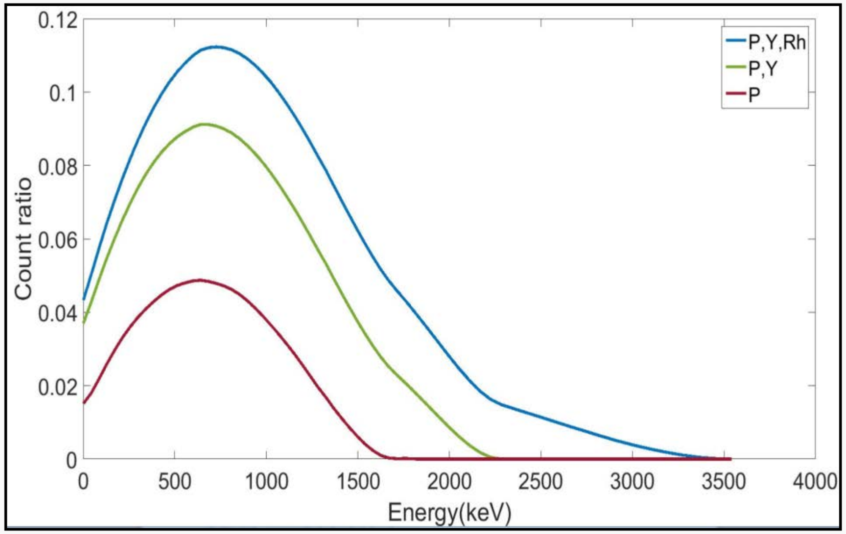

Fig. 5. The result of applying mixed beta spectrum separating method for mixed spectrum of ^{32}P, ^{90}Y and ^{106}Rh

Results

◆ **Result of separtion based on Spectrum Fitting Method**

Table 4: The application result of the proposed method to mixed spectrum data in various ratios of mixed nuclides

	Mixed Ratio (^{32}P: ^{90}Y: ^{106}Rh = 1:1:1)		
	Standard	Fitted	Error (%)
^{32}P	1	1.0053	0.5333
^{90}Y	1.3227	1.3169	0.4353
^{106}Rh	1.8648	1.8667	0.1013
	Mixed Ratio (^{32}P: ^{90}Y: ^{106}Rh = 2:2:1)		
	Standard	Fitted	Error (%)
^{32}P	1.2	1.2051	0.4250
^{90}Y	1.5872	1.5843	0.3684
^{106}Rh	1.1189	1.1200	0.1013

Results

Table 4 (cont.): The application result of the proposed method to mixed spectrum data in various ratios of mixed nuclides

	Mixed Ratio (^{32}P: ^{90}Y: ^{106}Rh = 1:2:2)		
	Standard	Fitted	Error (%)
^{32}P	0.6	0.6121	2.0138
^{90}Y	1.5872	1.5803	0.4353
^{106}Rh	2.2378	2.2401	0.1013
	Mixed Ratio (^{32}P: ^{90}Y: ^{106}Rh = 2:1:2)		
	Standard	Fitted	Error (%)
^{32}P	1.2	1.1988	0.0985
^{90}Y	0.7936	0.7891	0.5690
^{106}Rh	2.2378	2.2401	0.1013
	Mixed Ratio (^{32}P: ^{90}Y: ^{106}Rh = 2:1:1)		
	Standard	Fitted	Error (%)
^{32}P	1.5	1.4969	0.2069
^{90}Y	0.9920	0.9877	0.4353
^{106}Rh	1.3986	1.4000	0.1013

Results

Table 4 (cont.): The application result of the proposed method to mixed spectrum data in various ratios of mixed nuclides

	Mixed Ratio (^{32}P: ^{90}Y: ^{106}Rh = 1:2:1)		
	Standard	Fitted	Error (%)
^{32}P	0.75	0.7635	1.7970
^{90}Y	1.9840	1.9767	0.3684
^{106}Rh	1.3986	1.4000	0.1013
	Mixed Ratio (^{32}P: ^{90}Y: ^{106}Rh = 1:1:2)		
	Standard	Fitted	Error (%)
^{32}P	0.75	0.7556	0.7501
^{90}Y	0.9920	0.9863	0.5690
^{106}Rh	2.7972	2.8001	0.1013

Results

◆ Result of separtion based on Spectrum Fitting Method

- The Table 4 is the result of application of the proposed method to mixed spectrum data in various ratios of mixed nuclides.

- The value of ‘Standard’ is the value of **radioactivity** calculated by **interpolation**. This value is calculated by summing the number of count per specific energy in prepared interpolated data.

- The value of ‘Fitted’ is the radioactivity calculated by **fitting function**. The number of count per specific energy is calculated by multiplying the value of ratio of E_{max-10} and count ratio per energy.

- Error is a relative error between Standard and Fitted data. The error of ^{106}Rh is always same because the definition of ratio of ^{106}Rh is the first step of separation.

Results

◆ Result of separtion based on Spectrum Fitting Method

- The relative error of various ratios of mixed nuclides has minimum value as 0.0985 and maximum value as 2.013.

- As a result, this method is considered that can **separated mixed spectrum** accurately.

- Error of ^{32}P has the biggest value in overall ratio of mixed nuclides because imperfection of fitting function causes error and this value is **accumulated** during the separation process

- It is very easy and fast way to distinguish each nuclide's peak spectrums.

Conclusions

- A new method of analysis based curve **fitting** was proposed by improving the existing method for analyzing the multiple beta spectrums.
- It was checked that **sum of sine** fitting method is the best method for applying mixed spectrum data.
- The mixed spectrum of the three nuclides (^{32}P, ^{90}Y and ^{106}Rh) and several mixing ratio were used for the **evaluation** of the new method.
- It was shown that the performance of proposed beta spectrum analysis method was good at applying various ratios of mixed nuclides.
- In the further study, it was expected that the approach to more precise fitting method would be performed considering the **reduction of error.**

Thank you for your attention!